BI-OBJECTIVE OPTIMIZATION MODELS FOR GREEN VRP APPROACHES

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Abstract:

This research presents a mathematical programming formulation of green vehicle routing bi-criteria problems. The problems deal with G-VRP for routes crossing Navarre, Basque Country and La Rioja, Spain. The most significant bi-objectives for finding optimal routes are the minimization of the travelled total distance by the running vehicles versus the minimization of altitude differences within the route; the minimization of total distance versus the minimization of the carbon emission. Bi-Objective optimization models used mathematical programming formulations of multi traveling salesman problems. Constraints ensure that all vehicles begin and end their routes at the depot. The subtours solutions are going to be avoided. Weighted-Sum approach is implemented to solve multi-objective green vehicle routing optimization problems. The results of some computational experiments modeled after a real data from the Spanish food distribution company are reported.

Keywords:

Bi-Objective Decision Making, Green Logistics, Green Vehicle Routing, Mathematical Programming.

1. INTRODUCTION

This paper shows practical usefulness of bi-criteria mathematical programming for Green Vehicle Routing (G-VRP) for optimization of logistic services in the Spanish food distribution company. The results of some computational experiments modeled after a real data are reported.

Freight transportation planning includes many aspects, particularly when viewed from the multiple level of decision maker. The most famous problem at this level is Vehicle Routing Problem (VRP). VRP has been studied since 1959 with the objective to minimize the total distance traveled by all vehicles.

This paper shows practical usefulness of mathematical programming approaches to bi-objective optimization of green traveling salesman (TSP), transportation and vehicle routing problems (G-VRP) in the Spanish food distribution company. The results of some computational experiments modeled after a real data from a selected Spanish company are reported [1,2].
Traveling salesman problems (TSP) can be divided into two types: symmetric and asymmetric. Symmetric traveling salesman problems (STSP) require less computation processor units (CPU), in general. The asymmetric traveling salesman problems (ATSP) are more demanding from computational point of view. Most common generalization of TSP is multiple traveling salesman problems (mTSP), where more than one salesman allowed to be used in the solution [1,2].

In order to reduce the environmental impact from transportations, it is necessary to know the level of CO₂ emissions. The easier way to find this value is to firstly know the fuel consumption. Methodology for solving this problem and to determine the level of the fuel consumption can be found in many papers [2–7].

Altitude difference – the road gradient is important due to the fact, that the wheel horsepower demand increases on a slope and this will significantly affect fuel consumption. The size of the vehicle is also very important because the difference of fuel consumption from a small vehicle and a big one is remarkable.

Also the use of green freight corridors enables reductions in CO₂ emissions and in fuel consumption, and in the future should improve the environmental performance of road freight transport. For many years, the most important requirement has been to minimize the total distance travelled by vehicles or total time taken into the delivery.

It is only in the latest years when fuel consumption and CO₂ emissions have been taken into account as important issues. So after, they started to study a similar problem called Green VRP.

2. EROSKI GROUP

The largest part of Spain's powerful cooperative group Mondragon Corporación Cooperativa is the distribution arm, called Eroski Group [8]. The Eroski group is one of the leading chains of the Spanish retailing market. Currently Eroski is Spain’s fourth largest supermarket chain [9]. It operates more than 800 supermarkets throughout Spain. Under the name Eroski, besides supermarkets, there can also be find petrol stations and travel agencies. The company was founded in 1969 in the regions of Biscay and Basque Country in Spain as a co-operative between ten smaller consumer cooperatives in the region. Its headquarters located in Elorrio, Biscay. The name given, Eroski, is a combination of the Basque words “erosi” (to buy) and “toki” (place), which can be translated as “buying place” [8].

3. INPUT DATA FOR COMPUTATIONS

The real input data for computational experiments have been provided by the Spanish supermarket chain Eroski. The problem proposed is focused in the region of Navarra, Basque Country and La Rioja where the company has its sailing points and warehouses. The depot is Elorrio, located in Basque Country, is the first node of the transportation, from where all trucks are shipped to the other destinations, 27 of them will be studied here, most of them located in Navarra. One of the restrictions to take into account is that all locations can only be visited once, except the depot, as it is showed in models presented in this paper. It is important to know the difference of altitude in between two nodes of the route; this will give information of how is the road, if the gradient of the road is positive or negative. In case that it has negative values; the fuel consumption will be higher, and the CO₂ emissions, too. In this research the carbon dioxide emissions will be taken into account, because this will affect on the environmental costs of the company and surroundings.
In the following table (Table 1.), the names of all nodes where the transportation route has to go through are written in alphabetical order.

<table>
<thead>
<tr>
<th>Alegria-Dulantzi</th>
<th>Burlada</th>
<th>Logroño</th>
<th>Santa Cruz de Campezo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alsasua</td>
<td>Calahorra</td>
<td>Orkoien</td>
<td>Tafalla</td>
</tr>
<tr>
<td>Araia</td>
<td>Carcastillo</td>
<td>Pamplona</td>
<td>Tudela</td>
</tr>
<tr>
<td>Artajona</td>
<td>Cizur</td>
<td>Rada</td>
<td>Villaba</td>
</tr>
<tr>
<td>Autol</td>
<td>Estella</td>
<td>Salvatierra</td>
<td>Vitoria</td>
</tr>
<tr>
<td>Berriaín</td>
<td>Irurtzun</td>
<td>San Adrian</td>
<td></td>
</tr>
<tr>
<td>Berriozar</td>
<td>Lodosa</td>
<td>Sangüesa</td>
<td>Elorrio</td>
</tr>
</tbody>
</table>

The data provided by Spanish food distribution company shows the information of all orders and deliveries done in two months. The data includes the date of the orders and deliveries along with the amount of trucks used to carry out their orders. The capacity of the trucks it is showed in pallets and rolls, from here on it will be all the time measured in pallets.

In order to know the demand for each point, it is important to know the amount of supermarkets located in each location, because depending on it, one point will have more demand than others. In the data provided, this information can be obtained. It is showed the amount of pallets delivered to each point to each supermarket along with the date of order and delivery.

This data is determinant to make a decision on which truck to send, it will depend of how many pallets are needed.

4. PROBLEM FORMULATIONS, ALGORITHMS AND APPROACHES

Mathematical programming approaches deal with optimization problems of maximizing or minimizing a function of many variables subject to inequality and equality constraints and integrality restrictions on some or all of the variables.

Bi-objective models for transportation can be divided into three groups: transportation, traveling salesman and vehicle routing problems [2,6]. There are several ways to solve bi-objective transportation problems with environmental aspects [1,2,5,6,10,11,12,13].

Different algorithms have been considered to solve these types of aforementioned optimization problems [3,14]. Christofides [4] formulated optimization problem for transportation, which is referred to as the vehicle routing problem and is a generalization of the multiple travelling salesman problem.

5. OPTIMIZATION MODELS

Optimization problems are formulated as a bi-objective mixed integer program, which allows commercially available software (e.g. AMPL/CPLEX [15]) to be applied for solving practical instances.

Modifications of optimization models published by [7,16] for G-VRP are presented below. New models include not only reformulation of most constraints, but also bi-objective formulations and additional objectives, like altitude difference.
For optimization/minimization of CPU times, all computations have been done several times with use of different versions of subtours elimination constraints. Presented below constraint (10) is just one example of used subtours elimination.

Examples of Green Vehicle Routing (G-VRP) bi-objective model are formulated in subsections 5.1 and 5.2.

Notations for both models are presented in Table 2.

<table>
<thead>
<tr>
<th>Table 2 – Table of notations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Indices</strong></td>
</tr>
<tr>
<td>m – vehicles</td>
</tr>
<tr>
<td>n – nodes (depot and customers)</td>
</tr>
<tr>
<td><strong>Input parameters</strong></td>
</tr>
<tr>
<td>(\lambda) – weights for objectives - for altitude difference objective and for CO2 emission objective</td>
</tr>
<tr>
<td>(a_{ij}) – altitude difference between customer (node) (i) and customer (node) (j) [m]</td>
</tr>
<tr>
<td>(d_{ij}) – distance between customer (node) (i) and customer (node) (j) [km]</td>
</tr>
<tr>
<td>(e_{ij}) – (full load) – emission ration – preferably highest emission per km with full truck’s load [kg CO2/km]</td>
</tr>
<tr>
<td>(q_j) – load demanded by customer (node) (j) and load supplied by customer/depot (node) (j) [pallets]</td>
</tr>
<tr>
<td>(Q_k) – maximum capacity of load (cargo) in vehicle (k) [pallets]</td>
</tr>
<tr>
<td>(t_{ij}) – driving time/distance (average per km) between customer (node) (i) and (j) [km]</td>
</tr>
<tr>
<td>(T_k) – maximum allowable driving time/distance (based on average time per km/average speed in driving trucks)</td>
</tr>
<tr>
<td><strong>Decision variables</strong></td>
</tr>
<tr>
<td>(x_{ijk}) (\in) {0,1}, (i=1,...,n, j=1,...,n, k=1,...,m; i&lt;&gt;j) – if vehicle (k) visits customer (node) (j) immediately after customer (node) (i)</td>
</tr>
<tr>
<td>(y_{ik}) (\geq) 0, (i=1,...,n, k=1,...,m) – amount of goods shipped to customer (node) (i) in vehicle (k) [pallets]</td>
</tr>
</tbody>
</table>

### 5.1. Total distance vs. altitude differences

Bi-objective G-VRP model with minimization of total distance and minimization of altitude differences is formulated as follows (1):

Minimize

\[
\lambda \left( \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} d_{ij} x_{ijk} \right) + (1-\lambda) \left( \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} a_{ij} x_{ijk} \right)
\]

subject to

\[
\sum_{j=1}^{n} \sum_{k=1}^{m} x_{ijk} = m
\]

\[
\sum_{i=1}^{n} \sum_{k=1}^{m} x_{ijk} = m
\]

Constraints (2) and (3) ensure that all vehicles begin and end their routes at the depot.
\[ \sum_{k=1}^{m} x_{ijk} = 1, \quad i = 2, \ldots, n, \quad j = 2, \ldots, n \] (4)

Constraint (4) guarantees, that a single vehicle visits each node, except the depot.

\[ \sum_{j=1}^{n} x_{ijk} = 1, \quad j = 2, \ldots, n, \quad k = 1, \ldots, m \] (5)

\[ \sum_{i=1}^{n} x_{ijk} = 1, \quad i = 2, \ldots, n, \quad k = 1, \ldots, m \] (6)

Constraints (5) and (6) assure that each node, except the depot, is linked only with a pair of nodes, one preceding it and other following it.

\[ \sum_{j=2}^{n} q_j x_{ijk} \leq y_{ik}, \quad i = 1, \ldots, n, \quad k = 1, \ldots, m \] (7)

Constraint (7) is responsible for relation between variable \( x_{ijk} \) and \( y_{ik} \).

\[ \sum_{j=2}^{n} q_j x_{ijk} \leq Q_k, \quad k = 1, \ldots, m \] (8)

Constraint (8) ensures that no vehicle can be overloaded.

\[ \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} x_{ijk} \leq T_k, \quad k = 1, \ldots, m \] (9)

Constraint (9) does not permit that any vehicle exceeds the maximum allowable driving time/distance per day \( T_k \).

\[ \sum_{i \in S} \sum_{j \in S} x_{ijk} \geq 1, \quad S \subseteq \{1, \ldots, n\}, \quad k = 1, \ldots, m \] (10)

One example of subtours elimination constrains (10).

\[ x_{ijk} \in [0,1], \quad i = 1, \ldots, n, \quad j = 1, \ldots, n, \quad k = 1, \ldots, m \] (11)

\[ y_{ik} \geq 0, \quad i = 1, \ldots, n, \quad k = 1, \ldots, m \] (12)

Definitions of decision variable are presented in constraints (11) and (12).

5.2. Total distance vs. CO₂ emission

Bi-objective G-VRP model with minimization of total distance and minimization of CO₂ emission is formulated as follows (13):

Minimize

\[ \lambda \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} d_{ij} x_{ijk} + (1-\lambda) \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} e_{ij} x_{ijk} \] (13)

Subject to (2), (3), (4), (5), (6), (7), (8), (9), (10), (11) and (12).

Both models have been solved using Exact Methods.
6. COMPUTATIONAL RESULTS

In this section numerical examples and some computational results are presented to illustrate the solutions.

In the computational experiments the historical data is considered. Computational time takes only a fraction of a second to find optimal solution if any exists. The computational experiments have been performed using AMPL programming language [15] and the CPLEX.11 solver on a laptop with Intel® Core 2 Duo T9300 processor running at 2.5GHz and with 4GB RAM.

![Map of the depot and delivery points](image)

**Figure 1 – Map of the depot and delivery points**

In the Figure 1 different locations and the depot are presented, with the assigned number:
- For lambda=0.1 and lambda=0.25
- For lambda=0.5
- For lambda=0.75
- For lambda=0.9

Comparison of selected suboptimal route for Traveling Salesman Problem:
- Distance and Altitude: 1→6→10→8→5→2→9→7→4→3→1
- Distance and CO\(\text{2}\) emission: 1→6→2→8→9→5→10→4→3→7→1

7. CONCLUSIONS

Operations research techniques, tools and theories have long been applied to a wide range of issues and problems in green vehicle routing and transportation problems.

In this paper, we have discussed some green vehicle routing problems. The optimality criteria to be considered in suitable bi-criteria models were connected to environmental aspects. Thus, some mixed integer programming formulations of bi-criteria vehicle routing problems have been
considered. Some mathematical models were formulated under the assumption of existence of asymmetric distance-based costs and use of homogeneous fleet. The exact solution methods were applied in order to find optimal solutions. The software used to solve these models was the CPLEX solver with AMPL programming language.

Obtained results shows, that it is possible to control and limit the carbon emissions. There is a relation between obtained distances, driving times and altitude differences within obtained solutions versus obtained values of environmental objective. This relation proves the need for consideration of green aspects of transportation together in bi-criteria models.

The researchers were able to use real data from a Spanish company of groceries called Eroski. The solved problems deal with green logistics for routes crossing the Spanish regions of Navarre, Basque Country and La Rioja. The analyses of obtained results could help logistics managers to lead the initiative in area of green logistics by saving money paid as the direct cost of fuel and minimization of pollution. Regarding future work, we are currently working in an extended version of this problem, which includes heterogeneous fleet and combination of vehicle routing and portfolio problem.

8. ACKNOWLEDGEMENT

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9. REFERENCES


