MULTI-CRITERIA WAREHOUSES LOCATION PROBLEM IN THE LOGISTICS NETWORK

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Abstract:
This paper presents multi-criteria warehouses location problem in the logistics network. In order to solve this problem the location model of warehouse facilities within logistic network for manufacturing sector was developed. The limitations and optimization criteria of the model were determined. Optimization criteria refer to transport and storage costs, the total length of routes for vehicles carrying out the supplies, the land area for further expansion, the distance of warehouses from main transport routes. The final location of warehouse facilities was obtained using a point multi-criteria method.

Keywords:
multi-criteria warehouses location problem, a point multi-criteria method, logistic network

1. INTRODUCTION

The problem of warehouse situation is an issue that is generally known and widely discussed in the context of decision-making problems forming the logistical network of various types of enterprises. Depending on the complexity and structure of the network, the problem is being modified and acquires diverse forms. The classical issue of warehouse location is commonly defined in available literature as the capacitated warehouse location problem CWLP[1–3]. The structure of the logistic network consists of warehouse facilities and customers. The main objective is to find such a location of warehouse facilities that the costs arising from forwarding of a specific volume of goods to the customers are as low as possible. The transport cost depends on the volume of goods carried between warehouses and customers and also on the distance between them, and hence it is also necessary to set out volumes of carried goods. Limitations, on the other hand, arise from satisfying the needs of all customers and the capacitive limit for the dispatch of goods from the given warehouse facility. If no capacity limitation is imposed, we are faced with an uncapacitated warehouse location problem [4].

The basic stage in the configuration of logistic networks is the selection of locations of spot elements [5]. The logistical network is a set of elements (items): suppliers, manufacturers, wholesalers and recipients bound by diverse relations connected with the flow of materials. Depending on the number of intermediates on the transport route, the network structure may be single-level [3] or multi-level [6–10] which is called a hierarchical one. One of the characteristic features of the multi-level network is that materials have to flow from suppliers via subsequent
levels to recipients. In the multi-level network there are intermediaries, such as various types of warehouses, reloading terminals and logistic centres.

The problem of locating warehouse facilities is also studied in the context of planning the routes that vehicles have to cover. In this case it is defined in literature as warehouse location-routing problem [11,12]. The objective is to determine the location of warehouse facilities and goods delivery routes from warehouses to each customer, so that the cost arising from transport of a defined volume of goods to the customers is the lowest possible. This takes into consideration a situation in which the route of each vehicle contains a few unloading points (customers).

Furthermore, the problem of warehouse situation may be reviewed from two basic viewpoints. The first one is selecting the place where a new warehouse is to be erected [1,3,9] with respect to various viewpoints and the designed margin conditions. Basically in such a case potential locations for the construction of a new warehouse are indicated. The second aspect consists in the selection of warehouse locations from the already existing facilities. This implies a need for reconfiguration of the logistic network for the given sector, etc. [5,13].

In many cases, due to the multi-aspect nature of location problems, it is necessary to take into account optimising criteria that determine the choice of the best location solution. As regards the construction of a new warehouse, of particular importance are among others costs for land purchase for needs of construction works or the available land lot area. On the other hand, as regards restructuring of the logistic network, the selection of warehouses depends primarily on their distance from the remaining elements of the network, such as suppliers or recipients.

The warehouse location problem in the logistics network is multi-criteria optimization problem that depends on quantitative and qualitative criteria. In this issue the following criteria may be distinguished [9,14–19]:

- Costs: labour costs, transportation costs, storage costs, taxes. Labour costs change with respect to the life conditions at alternative locations. Transportation cost depends on the size of the cargo transported between facilities of the logistics network, distances between the warehouse and facilities within the network. Storage costs and taxes are different according to the regions.
- Labour characteristics: skilled labour and availability of labour force. This criterion determines the state of qualified labour at a given location. Skilled labour defines the personnel with appropriate qualities to perform work. The skilled labour and availability of labour force may be different according to the regions.
- Infrastructure: existence of modes of transportation, quality and reliability of modes of transportation. Existence of modes is understood as the availability of different transportation types in the given location e.g. railway. Quality and reliability of modes of transportation refers to timely deliveries.
- Market: proximity to customers, suppliers or producers. These factors in large degree influence on the transportation costs.
- Macro environment: policies of government e.g. tax exemptions.

The presented criteria emphasise the complexity of the problem arising from location of storage facilities and in a certain way determine the application of the multi-criteria decision-making assistance in selection of the optimum solution [9,15–19]. Multi-criteria decision assistance [20,21] is meant to provide the decision-maker with tools that allow solution of complex problems allowing for numerous viewpoints, as a rule quite contradictory.

The ultimate location of warehouse facilities arises from an assessment of diverse design variants executed using methods of multi-criteria assessment, such as for example the MAJA method [22]
or the spot method [9, 23]. Figure 1 presents a diagram of the decision-making process in warehouses selection.

![Diagram of decision-making process]

The article presents a location model for warehouse facilities for needs of handling a manufacturing plant taking into account four partial criteria. The model also takes into account the raw material flow in the logistic network on each working day of a manufacturing plant. It may also serve as a basis for the development of supply and receipt schedule of production raw materials, and in addition emphasises the practical aspect of the location problem subject to analysis. The final selection of warehouse location was done with the use of the spot method.

2. LOCATION MODEL OF WAREHOUSE FACILITIES WITHIN LOGISTIC NETWORK FOR MANUFACTURING SECTOR

2.1. General assumptions

The analysis comprises a logistic network of manufacturing plants. The network structure comprises the following: suppliers, warehouses and manufacturing plants. The suppliers may deliver raw materials directly or indirectly via warehouse facilities to manufacturing plants. The model assumes that the demand of recipients (manufacturing plants) is lower or equal to manufacturing capacity of the suppliers, and hence in each case the producers' demand would be met. Elements of structure in the analysed logistic network were presented on Figure 2.

The main objective is to determine the location of warehouse facilities from among those that are available in order to assure the best value of the adopted criterion function with concurrent satisfaction of needs of the recipients. The considered partial criteria comprised the minimum transport cost and raw materials passage through the warehouse, the minimum distance of supply execution to manufacturers, the maximum land area for further development, the minimum distance from the main transport routes. Data of the analysed case concern served manufacturing
plants, potential location points of warehouse facilities and transport links used for the implementation of supplies.

![Figure 2 – Elements in the logistic network of manufacturing plants](image)

The following definitions have been formulated for the needs of establishing a location model:

$V = \{ v; v = 1, 2, ..., v, ..., V \}$ – set of numbers of spot elements of the logistical network: suppliers, warehouses, manufacturing plants,

$T = \{ t; t = 1, 2, ..., t', ..., T \}$ – set of numbers of working days,

$DS = \{ v; a(v) = 0 \text{ for } v \in V \}$ – set of numbers of suppliers,

$MS = \{ v; a(v) = 1 \text{ for } v \in V \}$ – set of numbers of warehouses,

$P = \{ v; a(v) = 2 \text{ for } v \in V \}$ – set of numbers of manufacturing plants,

$D1 = [d1(v,v') \in R^+; v \in DS, v' \in MS]$ – distance matrices in relations: suppliers–warehouses,

$D2 = [d2(v,v') \in R^+; v \in DS, v' \in P]$ – distance matrices in relations: suppliers–enterprises,

$D3 = [d3(v,v') \in R^+; v \in MS, v' \in P]$ – distance matrices in relations: warehouses–enterprises,

$Q1 = [q1(v)]$ – vector of the volume of deliveries from suppliers,

$Q2 = [q2(v,t)]$ – matrix of the demand volume of enterprises on particular working days in pallet loading units,

$Q3 = [q3(v)]$ – vector of total demand of enterprises in pallet loading units,

$POJ = [poj(v)]$ – vector of warehouse capacity,

$K = [k(v)]$ – vector of passage costs of a load unit through the warehouse facilities,

$N = [n(v,v')]$ – matrix of the average pallet load units carried with one vehicle between particular points of the logistic network,

$LP = [lp(v)]$ – vector of additional land area for further development of warehouses in their particular locations,

$LK = [lk(v)]$ – vector of the distance of warehouses from the transport route,

$C = [c(v,v')]$ – matrix of transport costs of load unit per distance unit between particular facilities of the network.
2.2. Decision variables of model

Two types of decision–based variables were introduced, variables that define the volumes of cargo forwarding between particular facilities expressed in pallet cargo units and a binary variable that defines the selection of location for warehouse facilities. The variable that defines the volume of material stream flow between particular facilities serves the needs of determining the location of facilities from the viewpoint of minimum transport and storage costs and from the viewpoint of minimising the total length of supply routes. On the other hand, the binary variable was taken into account to delimit the location of warehouses with view to the maximum land area designated for further development and the minimum distance from main transport routes.

The first type of variable formulated as matrix $X_1$ (relation: suppliers – warehouses), $X_2$ (relation: suppliers – manufacturing plants), $X_3$ (relation: warehouses – manufacturing plants) on the interpretation of the volume of raw materials carried between network points on a given working day acquires the following form:

$$X_1 = \left[ x_1(v,v',t) : x_1(v,v',t) \in R^+ \cup \{0\}, v \in DS, v' \in MS, t \in T \right]$$  \hspace{1cm} (1)

$$X_2 = \left[ x_2(v,v',t) : x_2(v,v',t) \in R^+ \cup \{0\}, v \in DS, v' \in P, t \in T \right]$$  \hspace{1cm} (2)

$$X_3 = \left[ x_3(v,v',t) : x_3(v,v',t) \in R^+ \cup \{0\}, v \in MS, v' \in P, t \in T \right]$$  \hspace{1cm} (3)

The second type of variable assumes the following form:

$$Y = \{ y(v) : y(v) \in \{0,1\}, v \in MS \}$$  \hspace{1cm} (4)

Variable $y(v)$ assumes the value of 1 when the given warehouse is comprised by the logistic network of manufacturing plants and 0 if it is not.

2.3. Border conditions of location model

When formulating decision-making problems one of the main stages is to determine the system of limitations of the so-called border conditions. This is indispensable to determine the set of admissible solutions. To be able to achieve the solution, the set of admissible solutions has to be must at least contain a single element.

The analysed case has to take into account limitations arising from expectations of enterprises, their work pace and implementation of the manufacturing process, as well as limitations resulting from possibilities of the suppliers and also organisational and legal conditions of delivery implementation.

Main limitations of the model have been defined below.

1. Warehouses tend to have a limited capacity, and so the total of cargo volume supplied to each warehouse may not exceed its admissible capacity. Usage of the warehouse capacity depends on the volume of raw material delivered to the warehouse on the present working day and on the amount of raw material dispatched on the same day from the warehouse (first two elements of formula (5) and on the warehouse state on the preceding working days (element 3 and 4 from formula (5).

2. Maintaining the raw material flow stream through the warehouse, formula (6). Volume of raw material coming out from the given warehouse in a given working day may not exceed the current state of the warehouse on that day.
3. The entire cargo has to be collected from the supplier on a given working day, formula (7). The suppliers do not store the raw material. The limitation eliminates a situation in which a manufacturing plant does not use warehouses and picks up raw materials from the suppliers.

4. The demand of manufacturing plants in a given working day has to be met, formula (8).

5. Total demand of manufacturing plants has to be met, formula (9).

6. Limitations connected with the flow of cargo, i.e. the elimination of incorrect flows, non-negativity of flows, maintaining the flow volumes, excluding cyclical flows.

7. Keeping supply deadline in accordance with the adopted schedule.

Sample limitations in the mathematical form have been formulated in the following way:

\[ v' \in MS, t \in T \]
\[ \sum_{v \in DS} x_l(v, v', t) - \sum_{v \in P} x_3(v', v, t) + \sum_{t'=1}^{t-1} \sum_{v \in DS} x_l(v, v', t') - \sum_{t'=1}^{t-1} \sum_{v \in P} x_3(v', v, t') \leq \rho_0(v') \]  
(5)

\[ v' \in MS, t \in T \]
\[ \sum_{v \in DS} x_l(v, v', t) + \sum_{t'=1}^{t-1} \sum_{v \in DS} x_l(v, v', t') - \sum_{v \in P} x_3(v', v, t) + \sum_{t'=1}^{t-1} \sum_{v \in P} x_3(v', v, t') \geq \sum_{v \in P} x_3(v', v, t) \]  
(6)

\[ v \in DS, t \in T \]
\[ \sum_{v \in MS} x_l(v, v', t) + \sum_{v \in P} x_2(v, v', t) - q_1(v) = 0 \]  
(7)

\[ v' \in P, t \in T \]
\[ \sum_{v \in DS} x_2(v, v', t) + \sum_{v \in MS} x_3(v, v', t) - q_1(v) = q_2(v', t) \]  
(8)

\[ v' \in P \sum_{t \in T} \sum_{v \in DS} x_2(v, v', t) + \sum_{t \in T} \sum_{v \in MS} x_3(v, v', t) = q_3(v') \]  
(9)

### 2.4. Optimisation criteria

As has already been mentioned, selection of locations for warehouse facilities must take into account several aspects. The optimum configuration of the logistic network for needs of handling manufacturing plants is one that assures maximum benefits from the viewpoint of manufacturing plants (recipients) and minimum costs from the viewpoint of service providers. Consequently the defined configuration has to ensure obtaining services at an attractive price maintaining concurrently the appropriate quality of those services. The executed considerations suggest that seeking the best configuration of the logistic network should be based on several criteria – comprising different points of view as to the quality of solution – taken into account as partial assessment criteria. The vital partial assessment criteria, which should be taken into consideration for the assessment of admissible network configurations comprised the following:

1. minimum transport and storage costs,
2. minimising of the total length of routes for vehicles carrying out the supplies,
3. maximising the land area for further expansion,
4. minimising the distance of warehouses from main transport routes.

The formal partial criteria functions have been determined as follows:

### 1. minimum transport and storage costs:

\[ F_2(X1, X2, X3) = \]
\[ = \sum_{v \in DS} \sum_{v' \in MS \ t \in T} x_l(v, v', t) d_l(v, v') \cdot c(v, v') + \sum_{v \in DS} \sum_{v' \in P \ t \in T} x_2(v, v', t) d_2(v, v') \cdot c(v, v') + \]
\[ + \sum_{v \in MS} \sum_{v' \in P \ t \in T} x_3(v, v', t) d_3(v, v') \cdot c(v, v') + \sum_{v \in DS} \sum_{v' \in MS \ t \in T} x_l(v, v', t) k(v') \rightarrow \min \]  
(10)
2. minimising of the total length of routes for vehicles carrying out the supplies (the minimum routes entails a briefer supply implementation time; to allow the determination of the route length a calculation was made of the number of travels between particular points of the network):

\[
F_2(X_1, X_2, X_3) = \\
= \sum_{v \in DS} \sum_{v' \in MS} \sum_{t \in T} \left[ \frac{x_l(v, v', t)}{n(v, v')} \right] d_1(v, v') + \sum_{v \in DS} \sum_{v' \in PracT} \left[ \frac{x_2(v, v', t)}{n(v, v')} \right] d_2(v, v') + \\
+ \sum_{v \in MS} \sum_{v' \in PracT} \left[ \frac{x_3(v, v', t)}{n(v, v')} \right] d_3(v, v') \rightarrow \min
\]  

(11)

3. maximising the land area for further expansion:

\[
F_3(Y) = \sum_{v \in MS} y(v) \cdot l_p(v) \rightarrow \max
\]  

(12)

4. minimising the distance of warehouses from main transport routes:

\[
F_4(Y) = \sum_{v \in MS} y(v) \cdot l_k(v) \rightarrow \min
\]  

(13)

The determination of partial values of criteria is made for admissible solutions obtained taking into consideration limitations specified in item 2.3.

It is expected that the optimum configuration of the logistic network should allow achieving maximum benefits both to enterprises that make use of the planned network, as the handling companies. Consequently the objective is to find a compromise solution, i.e. optimum in the Pareto sense, and namely one for which values of all considered assessment criteria are the most advantageous (but not necessarily extreme ones). As is generally known, the implementation of such an approach is only possible thanks to multi-criteria optimising. The method of multi-criteria configuration assessment of the logistic network taking into account in the paper allows the determination of its best configuration pursuant to adopted criteria and providing for the decision-maker’s preferences.

3. MULTI-CRITERIA ASSESSMENT OF WAREHOUSE LOCATION

The diversity of optimisation criteria imposes the necessity of adopting the multi-criteria method to allow the selection of a warehouse location variant and taking into account weights for particular criteria. The final location of warehouse facilities may be obtained with the use of a point multi-criteria method. The algorithm of the point method along with the mathematical structure is available in [10, 19]. Particular stages of calculations carried out by the adopted algorithm of multi-criterial assessment may be presented in the following way (Figure 3):

a. Introduction of input data, based on which locations of warehouse facilities are set out.

b. Selection of an algorithm that sets out the admissible location variants for warehouse facilities.

The determination of admissible locations of warehouse facilities was executed with the LogMND [9] programme based on the Busacker-Gowen algorithm.

c. Determination of admissible location variants for warehouse facilities. In this stage of calculations a set of numbers is delimited for variants of preferred locations for warehouse facilities taking into consideration partial assessment criteria, which have been defined as:

\[
L = \{l : l = 1, 2, \ldots, L\}
\]  

(14)
d. Determination of matrices of assessments of warehouse location variants. For the admissible
variants of warehouse locations a determination is made of partial values of assessment
criteria, which in the subsequent stage of calculations are subjected to standardisation. The set
of partial criteria has been defined as follows:
\[ F = \{ f : f = 1, 2, \ldots, F \} \]  \hspace{1cm} (15)
where:
\( f = 1 \) – criterion of transport and warehousing costs [PLN], formula (10),
\( f = 2 \) – criterion of total length of routes over which goods are delivered [km], formula (11),
\( f = 3 \) – criterion of land designated for further development [m\(^2\)], formula (12),
\( f = 4 \) – criterion of warehouse distance from main transport routes [km], formula (13).

The matrix of location assessments has been defined as follows:
\[ X = \begin{bmatrix} x(f, l) & \cdots & x(f, l) \end{bmatrix} \quad f \in F, l \in L \]  \hspace{1cm} (16)
Standardisation of assessment values of preferred warehouse locations depends on the nature
of adopted sub-criteria. As regards the search for the maximum values of criteria, the standardised
assessment is determined according to formula (17), while as regards minimising of the criterion
based on the following formula (18):
\[ w(f, l) = \frac{x(f, l)}{\max_{l' \in L} \{ x(f, l') \}} \cdot 10 \]  \hspace{1cm} (17)
\[ w(f, l) = \frac{\min_{l' \in L} \{ x(f, l') \}}{x(f, l)} \cdot 10 \]  \hspace{1cm} (18)

e. The determination of relative partial values of assessment criteria \( c(f) \). It was adopted that
the relative weight of the given assessment is a figure within the range of \( <0, 100\%> \), with
the total of relative importance of all the considered assessment criteria equals to 100%.
f. Determination of assessment indices for each of the considered location variants \( W(l) \)
according to formula (19):
\[ \forall l \in LW(l) = \sum_{f \in F} c(f) \cdot w(f, l) \quad \forall l \in L \]  \hspace{1cm} (19)
The warehouse location variant with the highest value of the assessment index constitutes
the most advantageous solution.
The analysed logistic network consisted of suppliers, warehouse facilities and manufacturing plants. Three potential locations of warehouse facilities have been defined – $L_1$, $L_2$, $L_3$ (Figure 4), with the location of two of them in the logistic network being an optional one. This means that the logistic network may consist in one, two or three warehouses. The objective of the analysis is the determination of such locations of warehouse facilities in the analysed logistic network that the earlier adopted assessment criteria achieve concurrently the most beneficial of possible values.
Using the LogMND programme [9] five following variants were generated for the location of warehouses that satisfy the adopted limitations:

- variant 1 – location of warehouses L1 and L2,
- variant 2 – location of warehouse L2,
- variant 3 – location of warehouses L1, L2, L3,
- variant 4 – location of warehouses L3 and L2,
- variant 5 – location of warehouses L1 and L3.

Values of selection criteria for particular variants of warehouse locations and the weights of those criteria have been presented in Table 1. According to earlier remarks, minimising applies to the criteria of transport and storage criteria, total length of the route and distances of warehouses from main transport routes, while maximising is applied to the land area designated for further expansion.

| Assessment criterion of location variants \( f \) | Weight of criterion \( c(f) \) | Variant of warehouse location |
|---|---|---|---|---|---|---|---|
| 1 | 35 | \( l = 1 \) | 25 000 | 30 000 | 32 000 | 40 000 | 42 000 |
| 2 | 30 | \( l = 2 \) | 620 | 750 | 700 | 960 | 990 |
| 3 | 15 | \( l = 3 \) | 25 000 | 30 000 | 1500 | 3000 | 2000 |
| 4 | 20 | \( l = 4 \) | 30 | 50 | 40 | 60 | 25 |

Results of standardisation of variant assessments were presented in Table 2, while the assessment of warehouse location variants in Table 3.
Table 2 – Results of implemented standardisation of variant assessments

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Variant of warehouse location</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$l = 1$</td>
</tr>
<tr>
<td>1</td>
<td>10.00</td>
</tr>
<tr>
<td>2</td>
<td>10.00</td>
</tr>
<tr>
<td>3</td>
<td>8.33</td>
</tr>
<tr>
<td>4</td>
<td>8.33</td>
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Table 3 – Assessment index for adopted location variants

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Variant of warehouse location</th>
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<tbody>
<tr>
<td></td>
<td>$l = 1$</td>
</tr>
<tr>
<td>1</td>
<td>3.50</td>
</tr>
<tr>
<td>2</td>
<td>3.00</td>
</tr>
<tr>
<td>3</td>
<td>1.25</td>
</tr>
<tr>
<td>4</td>
<td>1.67</td>
</tr>
</tbody>
</table>

$W(l) = 9.42 \quad 7.90 \quad 7.39 \quad 6.46 \quad 6.96$

Table 3 shows that according to the determined weights, the most optimum location variants of those subjected to the analysis is variant 1. The major role in the selection of that location variant was played by criteria of costs and of the minimum length of forwarding job routes, in accordance with the adopted weights of decision-making entity’s propriety weights.

5. CONCLUSIONS

Location of warehouse facilities within the logistic network is a complex multi-criteria decision-making problem. The diversity of optimising criteria in the analysed problem imposes the necessity of applying in this case the above mentioned multi-criteria decision-making assistance to select the final location variant. As regards the single-criterion, the issue of particular importance in finding the optimum solution is adopting the appropriate optimising algorithms. Quite frequently in complex location problems use is made of heuristic algorithms. This type of algorithms does not assure an optimum solution in each and every case, frequently are limited to the determination of a sub-optimal solution. On the other hand, for the multi-criteria assessment of particular importance is the selection of criteria weights for variant appraisal.

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7. REFERENCES


